# Engineering Notes

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# **Fuel-Sensitivity Analyses for Jet** and Piston-Propeller Airplanes

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## Nomenclature

drag coefficient lift coefficient

specific fuel consumption based on weight flow

mean specific fuel consumption

D

 $\frac{E}{\bar{E}}$ lift-to-drag ratio, L/Dmean lift-to-drag ratio

 $P_T$ thrust power (product of T and V) fuel flow (weight per unit time)

Qrange

specific air range net installed thrust

true airspeed W aircraft weight empty airplane weight

fuel weight payload weight

horizontal distance coordinate nondimensional group of range terms

propeller efficiency

Subscripts

jet airplane

piston-propeller airplane p

start of cruise end of cruise

#### I. Introduction

N GENERAL, a change to the design of an airplane or a modification to an existing airplane may impact one or more of the following: 1) its weight, 2) its drag, and 3) its specific fuel consumption (SFC). The net effect on the airplane's fuel

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consumption resulting from a design change in which more than one of these parameters are altered can be difficult to assess without a detailed numerical model of the airplane's performance. Consider, for example, the incorporation of an active drag-reduction system (discussed in more detail in Sec. IV). Its function is to reduce drag, which increases the L/D ratio (and thus provides a performance benefit); however, the addition of the system increases the weight of the airplane (which results in a performance penalty), and the system needs power, which can be modeled as an increase in SFC. The net performance benefit (or penalty) may be quantified by the reduction (or increase) in the fuel consumed for a reference mission, and this is a function of the relative magnitudes of these three parameters. By considering small changes to the airplane's weight, drag, and SFC, the fundamental relationship linking these terms (and the resulting change to the fuel consumed during cruise) will be explored in this paper.

# II. Objectives

The primary objective was to develop a theoretical methodology to investigate the impact on fuel burn during cruise, for both jet and piston-propeller airplanes, arising from design changes that simultaneously alter the three governing performance parameters of an airplane in cruise: weight, drag, and SFC. The second objective was to evaluate the practicality of these equations by applying them to illustrative test cases.

# III. Theoretical Derivation

#### A. Basic Range Equations

The specific air range  $r_a$ , which is a measure of an airplane's efficiency, is the incremental still-air distance traveled (dx) per unit of fuel weight consumed  $(dW_f)$ ; that is,

$$r_a = -\frac{\mathrm{d}x}{\mathrm{d}W_f} = \frac{\mathrm{d}x/\mathrm{d}t}{-(\mathrm{d}W_f/\mathrm{d}t)} = \frac{V}{Q}$$
 (1)

where V is the true airspeed (TAS) and Q is the net fuel flow to the engine(s). A variable substitution is made by noting that the change in airplane weight (dW) is equal to the change in total onboard fuel weight  $(dW_f)$ . This is true for commercial airplane operations and for most general aviation airplane flights; changes of the airplane's weight that are *not* related to the fuel burned in the engine (e.g., loss of oil, water, or oxygen) are considered negligible. The still-air range R may thus be written as

$$R = \int_{\text{etert}}^{\text{end}} dx = -\int_{W}^{W_2} r_a \, dW = -\int_{W}^{W_2} \frac{V}{O} \, dW$$
 (2)

where  $W_1$  is the start-of-cruise weight and  $W_2$  is the end-of-cruise weight. For steady (i.e., unaccelerated) level flight, the fuel flow for a jet airplane is given by

$$Q = c_j T = c_j D = c_j \left(\frac{D}{L}\right) W = \frac{c_j}{E} W$$
 (3a)

and for a piston-propeller airplane, the fuel flow is given by

$$Q = c_p \frac{P_T}{\eta_p} = \frac{c_p DV}{\eta_P} = \frac{c_p WV}{\eta_P} \left(\frac{D}{L}\right) = \frac{c_p WV}{\eta_P E}$$
(3b)

where  $c_j$  and  $c_p$  are the specific fuel consumptions (of jet and piston-propeller airplanes, respectively), T is the thrust, D is the drag, L is the lift, E is the lift-to-drag ratio,  $P_T$  is the thrust power (product of T and V), and  $\eta_p$  is the propeller efficiency. Note that the SFC is defined in terms of weight flow rate (see [1] for equations in terms of mass flow rate, which are better suited to working with SI units).

Substituting Eqs. (3a) and (3b) into Eq. (2) yields the standard integral expressions for airplane range; that is, for a jet airplane,

$$R = -\int_{W}^{W_2} \frac{VE}{c \cdot W} \, \mathrm{d}W \tag{4a}$$

and for a piston-propeller airplane,

$$R = -\int_{W_1}^{W_2} \frac{\eta_p E}{c_p W} \, \mathrm{d}W \tag{4b}$$

#### B. Numerical Expression for Small Changes

The fuel used in cruise is defined as  $W_f$ . It thus follows that

$$W_f = W_1 - W_2 \tag{5}$$

If it is assumed that  $W_2$  (end-of-cruise weight),  $\bar{E}$  (mean lift-to-drag ratio during the cruise), and  $\bar{c}$  (mean SFC, either  $\bar{c}_j$  or  $\bar{c}_p$ ) are *independent* variables, then a linear expression for small changes of these variables may be formulated as follows:

$$\frac{\delta W_f}{W_2} = \left(\frac{\partial W_f}{\partial W_2}\right) \frac{\delta W_2}{W_2} + \left(\frac{\partial W_f}{\partial \bar{c}}\right) \frac{\delta \bar{c}}{W_2} + \left(\frac{\partial W_f}{\partial \bar{E}}\right) \frac{\delta \bar{E}}{W_2} \tag{6}$$

where  $\delta W_f/W_2$  represents a relative change in the fuel burn due to small changes in the variables  $W_2$ , SFC, and L/D. To determine expressions for the partial derivatives  $\partial W_f/\partial W_2$ ,  $\partial W_f/\partial \bar{c}$ , and  $\partial W_f/\partial \bar{E}$ , Eqs. (4a) and (4b) are used. If  $\bar{E}$  and  $\bar{c}$  are defined by

$$\bar{E} = -\frac{1}{W_f} \int_{W_1}^{W_2} E \, \mathrm{d}W \tag{7}$$

and

$$\bar{c} = -\frac{1}{W_f} \int_{W_1}^{W_2} c \, dW \tag{8}$$

respectively, then Eqs. (4a) and (4b) may be written for a jet airplane as

$$R = \frac{V\bar{E}}{\bar{c}_i} \ln\left(\frac{W_1}{W_2}\right) \tag{9a}$$

and for a piston-propeller airplane as

$$R = \frac{\eta_p \bar{E}}{\bar{c}_p} \ln \left( \frac{W_1}{W_2} \right) \tag{9b}$$

Equations (9a) and (9b) are, of course, forms of the Breguet range equation [1]. For this application, it is required that  $W_f$  be expressed in terms of R.

Substituting for  $W_1$  using Eq. (5) enables Eqs. (9a) and (9b) to be manipulated into the required format: that is, for a jet airplane,

$$W_f = (e^{\frac{R\tilde{c}_j}{V\tilde{E}}} - 1)W_2 \tag{10a}$$

and for a piston-propeller airplane,

$$W_f = (e^{\frac{R\bar{c}_p}{\eta_p E}} - 1)W_2 \tag{10b}$$

Partial derivatives of Eqs. (10a) and (10b) were then determined with respect to  $W_2$ ,  $\bar{E}$ , and  $\bar{c}$  and substituted into Eq. (6) to yield the following expressions. For a jet airplane,

$$\frac{\delta W_f}{W_2} = \left(e^{\frac{R\bar{c}_j}{V\bar{E}}} - 1\right) \frac{\delta W_2}{W_2} + \left(e^{\frac{R\bar{c}_j}{V\bar{E}}} \frac{R\bar{c}_j}{V\bar{E}}\right) \frac{\delta \bar{c}_j}{\bar{c}_i} - \left(e^{\frac{R\bar{c}_j}{V\bar{E}}} \frac{R\bar{c}_j}{V\bar{E}}\right) \frac{\delta \bar{E}}{\bar{E}} \quad (11a)$$

and for a piston-propeller airplane,

$$\frac{\delta W_f}{W_2} = (e^{\frac{R\bar{c}_p}{\eta_p \bar{E}}} - 1)\frac{\delta W_2}{W_2} + \left(e^{\frac{R\bar{c}_p}{\eta_p \bar{E}}}\frac{R\bar{c}_p}{\eta_p \bar{E}}\right)\frac{\delta \bar{c}_p}{\bar{c}_p} - \left(e^{\frac{R\bar{c}_p}{\eta_p \bar{E}}}\frac{R\bar{c}_p}{\eta_p \bar{E}}\right)\frac{\delta \bar{E}}{\bar{E}}$$
(11b)

It is noted that Eq. (11a) was previously presented by the author [2,3], but in a slightly different form: in [2,3], mass was used instead of weight and the SFC was based on the fuel mass flow rate. It is also acknowledged that the change in fuel burn due to a fixed change in aircraft weight [i.e., the first partial derivative in Eq. (11a)] is fundamentally the same as that presented earlier by Shustrov [4]. Similar sensitivity relationships for jet airplanes are given in [5]; however, they are not presented in the same condensed form, nor were they derived in the manner presented herein.

Because it is easier to comprehend a relative change in fuel (i.e.,  $\delta W_f/W_f$ ) than a ratio of the change in fuel to the end-of-cruise weight (i.e.,  $\delta W_f/W_2$ ), Eqs. (10a) and (10b) were substituted into Eqs. (11a) and (11b), respectively, to yield the final result:

$$\frac{\delta W_f}{W_f} = \frac{\delta W_2}{W_2} + \left(\frac{e^{\beta}\beta}{e^{\beta}-1}\right) \frac{\delta \bar{c}}{\bar{c}} - \left(\frac{e^{\beta}\beta}{e^{\beta}-1}\right) \frac{\delta \bar{E}}{\bar{E}}$$
(12)

where, for a jet airplane,

$$\beta = \frac{R\bar{c}_j}{V\bar{E}} \quad \text{and} \quad \bar{c} = \bar{c}_j$$

and for a piston-propeller airplane,

$$\beta = \frac{R\bar{c}_p}{\eta_p \bar{E}} \quad \text{and} \quad \bar{c} = \bar{c}_p$$

It is suggested that Eq. (12) is a convenient expression for conducting sensitivity analyses or trade studies, because the changes to  $W_2$ ,  $\bar{c}$ , and  $\bar{E}$  are expressed in relative terms. Furthermore,  $\beta$  is a dimensionless group of terms, which permits any consistent set of units to be used (as illustrated in Secs. IV and V). The impact on cruise fuel burn, which is a useful figure of merit to assess airplane economic performance, may be easily determined from percentage changes to the parameters  $W_2$ ,  $\bar{c}$ , and  $\bar{E}$ .

# IV. Case Study: Jet Airplane

#### A. Reference Airplane and Mission

The reference jet airplane was the B757-200. Because the performance characteristics of the numerical model do not exactly comply in all respects with those of the actual aircraft, a distinction is made herein, and the modeled version is referred to as the B757-200-class airplane.

The reference mission was a cruise of 2371 n mile at Mach 0.80 at 35,000 ft [International Standard Atmosphere (ISA) conditions]. The end-of-cruise mass  $W_2$  was used as a control parameter and set equal to 200,000 lb. Using the values for true airspeed, SFC, and lift-to-drag ratio, as given in Table 1, the start-of-cruise weight was determined using Eq. (9a) to be 242,000 lb.

#### **B.** Illustrative Application

Hybrid laminar flow control (HLFC) is an active drag-reduction technique that relies on 1) mechanical suction being applied to the leading 10–20% of the chord (i.e., ahead of the front spar) to stabilize the boundary layer and 2) a correctly profiled airfoil to generate a suitable pressure gradient that will maintain the laminar flow aft of the suction area. Laminar-turbulent transition is delayed by this technique, as described in [6–9], for example. The principle of HLFC can be applied to the wing, empennage, and engine nacelle.

The control of the boundary layer is achieved by sucking relatively small amounts of air through a perforated skin by means of a

Table 1 B757-200 class airplane data for reference condition (2371 n mile at Mach 0.80 at FL350)<sup>a</sup>

R	V	$ar{c}_j$	Ē	$W_1$	$W_2$	$W_f$
2371 n mile	461.1 kt	0.626 lb/h · lb	16.89	242,000 lb	200,000 lb	42,000 lb
4391 km	854.0 km/h	17.73 mg/s · N		109,770 kg	90,720 kg	19,050 kg

<sup>&</sup>lt;sup>a</sup>The preferred SI unit for SFC is defined in terms of *mass* flow rate [1] and is not dimensionally consistent with the derivations given in Sec. III.B, which are in terms of weight flow rate.

Table 2 GA airplane data for reference condition (518 n mile at 97 kt at sea level)<sup>a</sup>

Range	$ar{c}_j$	Ē	$\eta_{p}$	$W_1$	$W_2$	$W_f$
$3.15 \times 10^6 \text{ ft}$ 960 km	$1.847 \times 10^{-7} \text{ lbf} \cdot \text{s}^{-1}/\text{ft} \cdot \text{lbf} \cdot \text{s}^{-1}$ $6.18 \times 10^{-8} \text{ kg/s} \cdot \text{W}$	9.18	0.826	1874 lb 850 kg	1735 lb 787 kg	

<sup>&</sup>lt;sup>a</sup>The unit for SFC is given in a convenient form for use in Eq. (12). The preferred SI unit for SFC is defined in terms of *mass* flow rate [1] and is not dimensionally consistent with the derivations given in Sec. III.B, which are in terms of weight flow rate.

mechanical pump. The air is ducted beneath the skin through a network of chambers and pipes (which increases the airplane's weight) and is finally exhausted. Because power is required to drive the pump(s), the power offtake from the engine(s) must be increased; the degradation in engine performance can be represented as an SFC penalty.

#### C. Illustrative Use of a Small-Change Equation

Using the data given in Table 1, it follows from Eq. (12) that

$$\frac{\delta W_f}{W_f} = \frac{\delta W_2}{W_2} + 1.0983 \frac{\delta \bar{c}_j}{\bar{c}_j} - 1.0983 \frac{\delta \bar{E}}{\bar{E}}$$
 (13)

A nomogram (Fig. 1) was constructed using results obtained from Eq. (13). The illustrative lines in the figure show the case of a 1.5% change in end-of-cruise weight, a 2.0% change in SFC, and a 5% change in L/D. The net change in cruise fuel is -1.79%.

# V. Case Study: Piston-Propeller Airplane

#### A. Reference Airplane

The reference piston-propeller airplane is a two-seat General Aviation (GA) light aircraft, with a nonretractable landing gear. The mission was a cruise of 518 n mile at 97 kt at sea level (ISA conditions). The airplane data are given in Table 2.

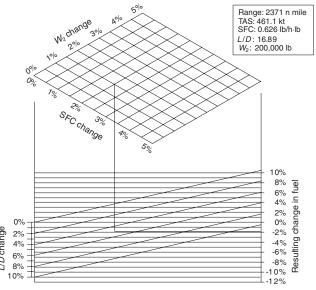


Fig. 1 Nomogram indicating fuel change with changing  $W_2$ , SFC, and L/D.

#### B. Application

Consideration is now given to the weight and drag changes associated with the installation of a *retractable* landing gear. In this case, there is no impact on the engine performance; hence, there is no associated change in the SFC.

Using the data given in Table 2, it follows from Eq. (11b) that

$$\frac{\delta W_f}{W_2} = 0.079749 \frac{\delta W_2}{W_2} - 0.082848 \frac{\delta \bar{E}}{\bar{E}}$$
 (14)

One of the ways that Eq. (14) can be used is illustrated in Fig. 2. The change in the cruise fuel weight  $\delta W_f$  was calculated corresponding to changes in the airplane's weight (ranging from 1 to 3%) and changes in the lift-to-drag ratio (ranging from 1 to 5%) for  $W_2=1735$  lb. If, for example, the retractable gear produced a 3% increase in the L/D ratio in cruise, then a weight increase of no more than 3% in the end-of-cruise weight is needed for a zero fuel penalty for the proposed design change. This may be converted to an empty airplane weight target, because the end-of-cruise weight  $W_2$  is the sum of the empty weight  $W_e$  and the payload weight  $W_{\rm pl}$ ; thus,

$$\frac{\delta W_e}{W_e} = \left(\frac{\delta W_2}{W_2}\right) \left(\frac{W_e + W_{\rm pl}}{W_e}\right) \tag{15}$$

# VI. Assessment of Independence of Variables

### A. Independence of Variables

It was assumed in Sec. III.B that  $W_2$  (end-of-cruise weight),  $\bar{c}$  (mean SFC), and  $\bar{E}$  (mean lift-to-drag ratio) are *independent* variables. This assumption permitted the formulation of Eqs. (11) and (12). In reality, the parameters are weakly linked (the nature of

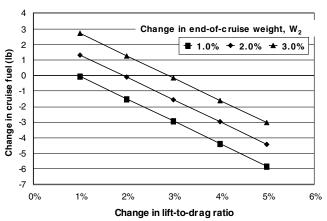


Fig. 2 Piston-propeller GA airplane: change in cruise fuel corresponding to changes in end-of-cruise weight and lift-to-drag ratio (518 n mile and 97 kt).

	Computer parameter: change in $W_2$								
	0%	1%	2%	3%	4%	5%			
Small-change equation									
$\Delta W_f/W_f$	0.0%	1.0%	2.0%	3.0%	4.0%	5.0%			
Computer model									
Run number	1	2	3	4	5	6			
$\bar{c}_j$ , lb/h · lb	0.6264	0.6263	0.6263	0.6264	0.6265	0.6268			
$ec{ar{E}}$	16.89	16.87	16.84	16.81	16.77	16.73			
$W_1$ , lb	242,000	244,467	246,934	249,464	251,995	254,548			
$W_2$ , lb	200,000	200,000	200,000	200,000	200,000	200,000			
$\Delta W_f/W_f$	0.00%	1.11%	2.22%	3.49%	4.75%	6.07%			

Table 3 Impact of small changes to end-of-cruise weight  $W_2$ 

Table 4 Impact of small changes to specific fuel consumption  $\bar{c}_i$ 

	Computer parameter: change in $\bar{c}_i$							
	0%	1%	2%	3%	4%	5%		
Small-change equation								
$\Delta W_f/W_f$	0.00%	1.10%	2.20%	3.30%	4.39%	5.49%		
Computer model								
Run number	1	2	3	4	5	6		
$\bar{c}_i$ , lb/h · lb	0.6264	0.6227	0.6390	0.6452	0.6515	0.6578		
$ar{c}_j$ , lb/h · lb $ar{E}$	16.89	16.89	16.88	16.88	16.87	16.87		
$W_1$ , lb	242,000	242,471	242,943	243,416	243,886	244,363		
$W_2$ , lb	200,000	200,000	200,000	200,000	200,000	200,000		
$\Delta W_f/W_f$	0.00%	1.12%	2.24%	3.37%	4.49%	5.63%		

Table 5 Impact of small changes to lift-to-drag ratio  $\bar{E}$ 

		Computer parameter: change in $\bar{E}$								
	0%	2%	4%	6%	8%	10%				
Small-change equati	on									
$\Delta W_f/W_f$	0.00%	-2.20%	-4.39%	-6.59%	-8.79%	-10.98%				
Computer model										
Run number	1	2	3	4	5	6				
$\bar{c}_j$ , lb/h · lb	0.6264	0.6269	0.6277	0.6486	0.6297	0.6310				
$ec{ar{E}}$	16.89	17.23	17.57	17.90	18.24	18.58				
$W_1$ , lb	242,000	241,138	240,325	239,561	238,838	238,152				
$W_2$ , lb	200,000	200,000	200,000	200,000	200,000	200,000				
$\Delta W_f/W_f$	0.00%	-2.05%	-3.99%	-5.81%	-7.53%	-9.16%				

which depends on the individual airplane's performance characteristics), as will be shown in this section.

#### **B.** Computer Performance Program

A spreadsheet-based computer program previously developed by the author [2,3,9] was used to accurately determine the fuel consumed by the reference jet airplane described in Sec. IV.A. The fuel consumed in cruise was found by a numerical integration of the range equation (4a). The approach was to divide the cruise into n intervals defined by the weight of the airplane at the station and to determine the specific air range  $r_a$  at each of the n+1 stations. Because the specific-air-range-vs-weight relationship is almost linear for small weight intervals, the trapezoidal rule was used to perform a numerical integration to yield the range; that is,

$$R = -\sum_{i=1}^{n} \left(\frac{r_{a_i} + r_{a_{i+1}}}{2}\right) (W_{i+1} - W_i)$$
 (16)

In this analysis, the SFC was not assumed to be constant. The specific air range was determined from velocity and fuel flow [see Eq. (1)], the latter term coming from lookup tables of corrected fuel flow vs thrust. The net engine thrust was set equal to the drag, which was determined from modeled drag polars for calculated values of the lift coefficient.

Using the computer program, conditions were evaluated at 21 stations during the cruise. Values of  $\bar{c}_j$  and  $\bar{E}$  were taken as the numerical averages of the SFC and L/D values calculated at these stations. These are, in fact, the values shown in Table 1 for the reference mission (see Sec. IV.A).

# C. Results from the Small-Change Equation

Relative changes to the fuel mass  $\delta W_f/W_f$  for the reference mission (see Sec. IV.A) were calculated using Eq. (12) with the numerical values as indicated in Eq. (13) by individually considering small changes to  $W_2$ ,  $\bar{c}_j$ , and  $\bar{E}$ . For each set of calculations, only one parameter was incrementally changed and the other two were held constant. The results are given in the first rows of Table 3 (where  $W_2$  varied from 0 to 5%), Table 4 (where  $\bar{c}_j$  varied from 0 to 5%), and Table 5 (where  $\bar{E}$  varied from 0 to 10%).

#### D. Results from the Computer Model

The results of the three sets of calculations, performed using the computer program, are given in Tables 3–5. In each case, one governing parameter was incrementally changed, whereas the other two were allowed to vary, according to the underlying mathematical relationships.

1) For the end-of-cruise-weight sensitivity study (Table 3), the value of  $W_2$  was changed in the computer program and the impact on

the cruise fuel  $W_f$  was determined. The results are for six computer runs, with run 1 corresponding to the reference (baseline) mission and runs 2 to 6 corresponding to increases in  $W_2$  in steps of 1%. Each time, the change in  $W_f$  was calculated as a ratio of the baseline value of  $W_f$ .

- 2) For the SFC sensitivity study (Table 4), the *fuel flow* was factored within the engine database to give the required step changes in SFC. The mean SFC  $\bar{c}_j$  for each computer run was calculated and compared with the baseline condition (i.e., run 1). Because the range did not change, the start-of-cruise weight  $W_1$  increased by a small amount due to the increased fuel load. This meant that the thrust also increased a little in response to the increased drag.
- 3) For the lift-to-drag-ratio sensitivity study (Table 5), the *drag coefficient* was reduced within the program database. The mean lift-to-drag ratio  $\bar{E}$  was then calculated and the percentage change was determined with respect to the baseline condition (i.e., run 1). In this case, the thrust reduced due the lower drag and lower weight.

#### E. Discussion of Results

The results given by Eq. (12) (first row in Tables 3–5) were compared with the results obtained from the computer model (last row in Tables 3–5), which took into account the inherent coupling between the variables  $W_2$ ,  $\bar{c}_j$ , and  $\bar{E}$ . The following was noted:

- 1) For a 5% increase in  $W_2$ , Eq. (12) yielded a 5.0% increase in  $W_f$ . This compared reasonably well with the 6.07% change obtained using the computer model (Table 3). It is evident that as the end-of-cruise weight increases,  $C_L$  and  $C_D$  would both increase during the cruise, but not necessarily by the same amount. At the higher  $C_L$  values, the L/D ratio is seen to be a little lower for this particular cruise data; however, the impact is small. Additionally, the higher drag would require a proportionally greater thrust, and this would, in general, change the SFC. For this particular model, the mean SFC varies nonlinearly by a very small amount, with an increasing value of  $W_2$ .
- 2) For a 5% increase in SFC, Eq. (12) yielded a 5.49% increase in  $W_f$ . This compared very well with the 5.63% change obtained using the computer model. Because the weight increase due to the increased fuel requirement is small, the mean L/D ratio does not change very much.
- 3) For a 10% increase in L/D, Eq. (12) yielded a -9.16% change in  $W_f$ . This compared reasonably well with the -10.98% change obtained using the computer program. With reduced fuel load due to the lower drag, the airplane is lighter at all times during the cruise. This will affect the SFC at each point during the cruise. The mean SFC increases in this case; however, due to the nonlinearity of SFC

variation with thrust, this cannot be regarded as a general observation.

#### VII. Conclusions

- 1) The mathematical formulation herein derived [and encapsulated in Eq. (12)] is a convenient expression for rapidly assessing proposed airplane design changes as part of a sensitivity or trade study in which the envisaged changes simultaneously alter the airplane's weight, specific fuel consumption, and drag.
- 2) The impact on cruise fuel burn may be satisfactorily estimated from relative changes to  $W_2$  (end-of-cruise weight),  $\bar{c}$  (mean SFC), and  $\bar{E}$  (mean lift-to-drag ratio) using Eq. (12).
- 3) The underlying assumption of independence of the governing parameters (i.e.,  $W_2$ ,  $\bar{c}$ , and  $\bar{E}$ ) was explored for a constant height cruise using a computer model of an airplane in the class of the B757-200. The results indicated that for a jet airplane, the parameters are only weakly linked, the nature of which depends on the airplane's performance characteristics.

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